CLAIMS

What is claimed is:

1	1.	A metl	nod for determining forces to be applied to a user through a haptic
2	interface, said method comprising the steps of:		
3		(a)	generating a representation of an object in graphic space;
4		(b)	sensing a position of a user in real space;
5		(c)	determining a haptic interface location in graphic space in response to
6			said position of said user in real space;
7		(d)	determining a fiducial object location in graphic space; and
8		(e)	calculating a stiffness force to be applied to said user in real space in
9			response to said haptic interface location and said fiducial object
10			location in graphic space.
1	2.	The m	nethod of claim 1 wherein the haptic interface is a point and wherein the
2	fiducial object is a point.		
1	3.	The m	nethod of claim 1 wherein the step of calculating a stiffness force further
2	comp	rises the	step of:
3			determining a displacement between said fiducial object location and
4		said h	naptic interface location; and
5			calculating said stiffness force in response to said displacement.
1	4.	The n	nethod of claim 1 wherein said haptic interface has a velocity and said
2	fiduc	ial objec	t has a velocity, said method further comprising the steps of:
3		(f)	determining the velocity of said haptic interface relative to a common
4			reference;
5		(g)	determining the velocity of said fiducial object relative to said common
6			reference;

7	(h)	calculating a damping force to be applied to said user in real space in	
8		response to said velocity of said haptic interface and said velocity of said	
9		fiducial object; and	
10	(i)	calculating a feedback force to be applied to said user in real space in	
11		response to said stiffness force and said damping force.	
		nethod of claim 4 wherein said common reference is said representation of	
1			
2	said object in	graphic space.	
1	6. The m	nethod of claim 4 wherein the step of generating a representation of an	
2	object in grap	hic space further comprises the step of defining said object as a mesh of	
3		s, each of said planar surfaces having nodes 1-n, wherein n is a positive	
4	integer.		
1		nethod of claim 4 wherein the step of generating a representation of an	
2	object in graphic space further comprises the step of defining said object as an n-noded		
3	polygon havir	ng nodes 1-n, wherein n is a positive integer.	
1	8. The n	nethod of claim 6 wherein said planar surfaces are triangular, said	
2		nar surfaces having a first node, a second node and a third node.	
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1	9. The r	nethod of claim 8 wherein the stiffness force has a magnitude and wherein	
2	the step of ca	lculating a stiffness force to be applied to said user in real space further	
3	comprises the	e step of:	
4		calculating the magnitude of the stiffness force, wherein the step of	
5	calcu	lating the magnitude of the stiffness force further comprises the steps of:	
6		associating a stiffness value with said first node, said second	
7		node and said third node of each said triangular planar surface;	
8		determining on which of said triangular planar surfaces said	
9		fiducial object is located; and	

10	computing a stiffness value of said fiducial object location by
11	interpolating said stiffness values associated with said first node, said
12	second node and said third node of each of said triangular planar
13	surfaces on which said fiducial object is located.
1	10. The method of claim 8 wherein the stiffness force has a direction and wherein
2	the step of calculating a stiffness force to be applied to said user in real space further
3	comprises the step of:
4	calculating the direction of the stiffness force, wherein the step of
5	calculating the direction of the stiffness force further comprises the steps of:
6	associating a surface normal with each said first node, said
7	second node and said third node of each said triangular planar surface;
8	determining on which of said triangular planar surfaces said
9	fiducial object is located; and
10	computing a surface normal for said fiducial object location by
11	interpolating said surface normals associated with said first node, said
12	second node and said third node of each of said triangular planar
13	surfaces on which said fiducial object is located.
1	11. The method of claim 8 wherein the damping force has a magnitude and wherein
2	the step of calculating a damping force to be applied to said user in real space further
3	comprises the step of:
4	calculating the magnitude of the damping force, wherein the step of
5	calculating the magnitude of the damping force further comprises the steps of:
6	associating a damping coefficient with each said first node, said
7	second node and said third node of each said triangular planar surface;
	determining on which of said planar surfaces said fiducial objec
8	
9	is located; and computing a damping coefficient of said fiducial object location
10	• -
11	by interpolating said damping coefficients associated with said first

12	node, said second node and said third node of each of said triangular	
13	planar surfaces on which said fiducial object is located.	
1	12. The method of claim 8 wherein the damping force has a direction and wherein	
2	the step of calculating a damping force to be applied to said user in real space further	
3	comprises the step of:	
4	calculating the direction of the damping force, wherein the step of	
5	calculating the direction of the damping force further comprises the steps of:	
6	associating a surface normal with each said first node, said	
7	second node and said third node of each said triangular planar surface;	
8	determining on which of said triangular planar surfaces said	
9	fiducial object is located; and	
10	computing a surface normal for said fiducial object location by	
11	interpolating said surface normals associated with said first node, said	
12	second node and said third node of each said triangular planar surfaces	
13	on which said fiducial object is located.	
1	13. The method of claim 6 wherein the stiffness force has a magnitude and wherein	
2	the step of calculating a stiffness force to be applied to said user in real space further	
3	comprises the step of:	
4	calculating the magnitude of the stiffness force, wherein the step of	
5	calculating the magnitude of the stiffness force further comprises the steps of:	
6	associating a stiffness value with each said 1-n nodes of each	
7	said planar surface;	
8	determining on which of said planar surfaces said fiducial object	ct
9	is located; and	
10	computing a stiffness value of said fiducial object location by	
11	interpolating said stiffness values associated with said 1-n nodes of eac	h
12	of said planar surfaces on which said fiducial object is located.	

l	14.	The method of claim 6 wherein the stiffness force has a direction and wherein		
2	the step of calculating a stiffness force to be applied to said user in real space further			
3	compr	comprises the step of:		
4		calculating the direction of the stiffness force, wherein the step of		
5		calculating the direction of the stiffness force further comprises the steps of:		
6		associating a surface normal with each said 1-n nodes of each		
7		said planar surface;		
8		determining on which of said planar surfaces said fiducial object		
9		is located; and		
10		computing a surface normal for said fiducial object location by		
11		interpolating said surface normals associated with said 1-n nodes of each		
12		of said planar surfaces on which said fiducial object is located.		
1	15.	The method of claim 6 wherein the damping force has a magnitude and wherein		
2	the step of calculating a damping force to be applied to said user in real space further			
3	comprises the step of:			
4		calculating the magnitude of the damping force, wherein the step of		
5		calculating the magnitude of the damping force further comprises the steps of:		
6		associating a damping coefficient with each said 1-n nodes of		
7		each said planar surface;		
8		determining on which of said planar surfaces said fiducial object		
9		is located; and		
10		computing a damping coefficient of said fiducial object location		
11		by interpolating said damping coefficients associated with said 1-n		
12		nodes of each of said planar surfaces on which said fiducial object is		
13		located.		
1	16.	The method of claim 6 wherein the damping force has a direction and wherein		
2	the s	tep of calculating a damping force to be applied to said user in real space further		
2	comprises the step of:			

4	calculating the direction of the damping force, wherein the step of
5	calculating the direction of the damping force further comprises the steps of:
6	associating a surface normal with each said 1-n nodes of each
7	said planar surface;
8	determining on which of said planar surfaces said fiducial object
9	is located; and
10	computing a surface normal for said fiducial object location by
11	interpolating said surface normals associated with said 1-n nodes of each
12	said planar surfaces on which said fiducial object is located.
1	17. The method of claim 4 wherein said object has a surface and wherein the step of
2	generating a representation of an object in graphic space further comprises the steps of:
3	describing the surface of said object using a coordinate system, said
4	coordinate system comprising a plurality of coordinate sets; and
5	associating a parameter with each set of coordinates of said plurality of
6	coordinate sets.
1	18. The method of claim 17 wherein the parameter is a stiffness value and wherein
2	the step of calculating a stiffness force to be applied to said user in real space further
3	comprises the steps of:
4	determining which set of coordinates of said plurality of coordinate sets
5	describes said fiducial object location; and
6	determining a stiffness value of said fiducial object location in response
7	to said set of coordinates describing said fiducial object location.
1	19. The method of claim 17 wherein the parameter is a surface normal and wherein
2	the step of calculating a stiffness force to be applied to said user in real space further
3	comprises the steps of:
4	determining which set of coordinates of said plurality of coordinate sets
5	describes said fiducial object location; and

6	determining a surface normal for said fiducial object location in		
7	response to said set of coordinates describing said fiducial object location.		
1	20. The method of claim 17 wherein the parameter is a surface normal and wherein		
2	the step of calculating a damping force to be applied to said user in real space further		
3	comprises the steps of:		
4	determining which set of coordinates of said plurality of coordinate se		
5	describes said fiducial object location; and		
6	determining a surface normal for said fiducial object location in		
7	response to said set of coordinates describing said fiducial object location.		
1	21. The method of claim 17 wherein the parameter is a displacement force, wherein		
2	each set of coordinates has a corresponding surface normal and wherein the step of		
3	calculating a damping force to be applied to said user in real space further comprises the		
4	step of:		
5	adding each said displacement force of each said set of coordinates to		
6	said corresponding surface normal to calculate a total surface normal for each		
7	said set of coordinates;		
8	determining which set of coordinates of said plurality of coordinate set		
9	describes said fiducial object location; and		
10	determining a total surface normal for said fiducial object location in		
11	response to said set of coordinates describing said fiducial object location.		
1	22. The method of claim 4 further comprising the step of:		
2	subsequent to calculating said feedback force to be applied to said user		
3	in real space, producing said feedback force by a force actuating device.		
1	23. The method of claim 22 wherein said object has a surface and wherein said		
2	generated representation of said object includes a surface representation, said method		
3	further comprising the steps of:		

4		defining a plane tangent to said surface representation at said fiducial
5		object location,
6		wherein said produced feedback force is normal to said plane.
1	24.	The method of claim 22 wherein said velocity of said fiducial object has a first
2	direct	ion and wherein said producèd feedback force has a second direction, said second
3	direct	ion being opposite said first direction.
1	25.	The method of claim 4 further comprising the steps of:
2		determining a first stiction location in graphic space;
3		calculating a friction force to be applied to said user in real space in
4		response to said haptic interface location and said first stiction location in
5		graphic space; and
6		calculating a total force to be applied to said user in real space in
7		response to said friction force and said feedback force.
1	26.	The method of claim 25 further comprising after the step of calculating a friction
2	force	and before the step of calculating a total force to be applied to said user in real
3	space	, the steps of:
4		multiplying said feedback force by a first coefficient of friction to
5		determine a product;
6		determining whether said friction force is greater than or equal to said
7		product of said feedback force and said first coefficient of friction;
8		updating said first stiction location to a second stiction location in
9		graphic space if said friction force is greater than said product; and
10		re-calculating said friction force in response to said second stiction
11		location in graphic space if said first stiction location was updated.
ı	27.	The method of claim 26 wherein the step of updating said first stiction location
2	toas	econd stiction location in graphic space further comprises the steps of:

3	calculating a maximum friction force in response to said normal force to
4	be applied to said user in real space and a second coefficient of friction;
5	placing said second stiction location along a line intersecting said first
6	stiction location and a current haptic interface location; and
7	placing said second stiction location along said line at a distance from
8	said current haptic interface location where said friction force to be applied to
9	said user is substantially equal to said calculated maximum friction force.
1	28. The method claim 27 wherein said first coefficient of friction is substantially
2	equal to said second coefficient of friction.
1	29. The method of claim 27 wherein said second coefficient of friction is lower than
2	said first coefficient of friction.
1	30. The method of claim 25 further comprising after the step of calculating a total
2	force to be applied to said user in real space, the steps of:
3	calculating a first distance in graphic space between said fiducial object
4	location and said first stiction location;
5	calculating a second distance in graphic space between said haptic
6	interface location and said fiducial object location;
7	determining whether said first distance is greater than said second
8	distance multiplied by a third coefficient of friction;
9	if said first distance is greater than said second distance multiplied by
10	said third coefficient of friction, moving said first stiction location in graphic
11	space toward said fiducial object location until said first distance is not greater
12	than said second distance multiplied by said third coefficient of friction; and
13	calculating a change in total force to be applied to said user in real space
14	in response to said moved first stiction location and said haptic interface
15	location.

1	31.	The m	nethod of claim 4 further comprising the steps of:
2			determining a first stiction location in graphic space, said first stiction
3		location	on having a velocity;
4			calculating a friction force to be applied to said user in real space in
5		respo	nse to said fiducial object location, said first stiction location, said velocity
6		of said	d fiducial object and said velocity of said first stiction location; and
7			calculating a total force to be applied to said user in real space in
8		respo	nse to said friction force and said feedback force.
1	32.	A me	thod for determining forces to be applied to a user through a haptic
2	interfa	ace, said	method comprising the steps of:
3		(a)	generating a representation of an object in graphic space;
4		(b)	displaying said representation of said object on a display;
5		(c)	sensing a first position of a user in real space;
6		(d)	determining a first haptic interface location in graphic space in response
7			to said first position of said user in real space;
8		(e)	determining a first fiducial object location in graphic space;
9		(f)	displaying said first position of said user relative to said object on said
10			display; and
11		(g)	calculating a first stiffness force to be applied to said user in real space
12			in response to said first haptic interface location and said first fiducial
13			object location in graphic space.
1	33.	The r	nethod of claim 32 further comprising the steps of:
2		(h)	storing a first set of state variables representing said first fiducial object
3			location in graphic space;
4		(i)	sensing a second position of said user in real space;
5		(j)	determining a second haptic interface location in graphic space in
6			response to said second position of said user in real space:

7	(k)	calculating a second fiducial object location in graphic space in response
8		to said first set of state variables and said second haptic interface
9		location;
10	(1)	removing said first position of said user relative to said object from said
11		display and displaying said second position of said user relative to said
12		object on said display; and
13	(m)	repeating step (g) for said second haptic interface location and said
14		second fiducial object location to determine a second stiffness force to
15		be applied to said user in real space.
1	34. The r	nethod of claim 33 wherein the step of generating a representation of an
2	object in grap	shic space further comprises the step of defining said object as a mesh of
3	surfaces, and	wherein the step of calculating a second fiducial object location in graphic
4	space further	comprises the steps of:
5	(n)	determining a line intersecting said first fiducial object location and said
6		second haptic interface location;
7	(o)	determining which of said surfaces said line intersects;
8	(p)	determining whether each said surface intersected by said line is located
9		a negative distance from said first fiducial object location;
10	(q)	determining whether each said surface intersected by said line is located
11		a positive distance from said second haptic interface location;
12	(r)	defining each said surface intersected by said line, located a negative
13		distance from said first fiducial object location and located a positive
14		distance from said second haptic interface location as an active
15		constraint;
16	(s)	computing said second fiducial object location in response to said
17		second haptic interface location and said active constraints;

18	(t)	repeating steps (n)-(r) using said computed second fiducial object
19		location in place of said second haptic interface location to determine
20		additional active constraints;
21	(u)	re-computing said second fiducial object location in response to said
22		additional active constraints; and
23	(v)	repeating steps (t)-(u) until no new additional active constraints are
24		determined.
1	35. The m	nethod of claim 33 wherein the step of generating a representation of an
2	object in grap	hic space further comprises the step of defining said object as a mesh of
3	triangular plas	nar surfaces, each of said triangular planar surface having three boundary
4	lines, and who	erein the step of calculating a second fiducial object location in graphic
5	space further	comprises the steps of:
6	(n)	determining a line intersecting said first fiducial object location and said
7		second haptic interface location;
8	(o)	determining which of said triangular planar surfaces said line intersects;
9	(p)	determining whether each said triangular planar surface intersected by
10		said line is located a negative distance from said first fiducial object
11		location;
12	(q)	determining whether each said triangular planar surface intersected by
13		said line is located a positive distance from said second haptic interface
14		location;
15	(r)	defining each said triangular planar surface intersected by said line,
16		located a negative distance from said first fiducial object location and
17		located a positive distance from said second haptic interface location as
18		an active constraint;
19	(s)	computing said second fiducial object location in response to said
20		second haptic interface location and said active constraints;

21	(t)	repeating steps (n)-(r) using said computed second fiducial object
22		location in place of said second haptic interface location to determine
23		additional active constraints;
24	(u)	re-computing said second fiducial object location in response to said
25		additional active constraints; and
26	(v)	repeating steps (t)-(u) until no new additional active constraints are
27		determined.
1	36. An app	paratus for determining and applying feedback forces to a user in real
2	space through a haptic interface, said apparatus comprising:	
3		a sensor sensing positions of a user in real space;
4		a first processor in electrical communication with said sensor executing
5	an alg	orithm to determine feedback forces to be applied to said user in real
6	space,	said algorithm including:
7		a module generating a representation of an object in graphic
8		space,
9		a module determining a haptic interface location in graphic
10	space	in response to a position of said user in real space,
11		a module determining a fiducial object location in graphic
12		space, and
13		a module calculating a stiffness force to be applied to said user
14		in real space in response to said haptic interface location and said
15		fiducial object location;
16		a display processor in electrical communication with said first processor
17	said d	lisplay processor displaying said representation of said object on a display
18	and d	isplaying said positions of said user relative to said object on said display;
19	and	

20	a force actuator in electrical communication with said first processor and	
21	said display processor, said force actuator producing said stiffness force to be	
22	applied to said user in real space.	
1	37. The apparatus of claim 36 wherein said first processor and said display	
2	processor are the same processor.	
1	38. The apparatus of claim 36 wherein said module generating a representation of an	
2	object in graphic space, said module determining a haptic interface location in graphic	
3	space, said module determining a fiducial object location in graphic space and said	
4	module calculating a stiffness force to be applied to said user in real space are separate	
5	devices.	